

An Analysis of the Schrodinger Equation Model for the Distribution Rate of Stock Returns

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ABSTRACT

Quantum mechanics is a theory that describes the behavior of particles in the microscopic world. If the stock index can be considered an object on a macro scale, then every stock of a stock index is an object on a micro-scale. The stock price can be analogous to being a particle. This study aimed to obtain the density distribution of stock returns. Modeling stock returns distribution using a Schrodinger equation model with the assumption that stock is a particle in the good delta potential function so that stock returns as analogous to particles can be known. The Schrodinger equation can calculate stock returns expressed as an exponential distribution. The stock return density distribution using Schrodinger equation model has a higher kurtosis value than the kurtosis value in the normal distribution. The kurtosis value is the degree of the peak height of a distribution. The stock price data used is the stock price data of PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. during 2013-2018. This study shows the stock price of PT. Unilever Indonesia Tbk. has a more stable average stock price return with a more negligible risk of loss than the stock price of PT. United Tractors Tbk.

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1. INTRODUCTION

The capital market has a vital role in influencing economic growth by encouraging people to save and providing financial assistance to a company. A capital market is a meeting place between sellers and buyers, with the object being traded in the form of securities. One of the securities instruments is shares. Shares are securities as proof of ownership of a company or a sign of equity participation in the company. Stocks have a price value that fluctuates every time in the stock market. Stock price fluctuations are influenced by the amount of supply and demand on the stock market. If the number of offers exceeds the number of requests, the stock price will fall. Vice versa, the stock price will rise if the number of requests exceeds the number of offers. These stock price fluctuations affect the amount of profit that investors get in the form of dividends and capital gains and the amount of risk that investors get in the form of capital loss (Cotfas, 2013; Sukmana et al., 2019; Indryani et al., 2021).

Stock price predictions are necessary for investors to maximize profits and minimize losses. Two types of analysis can be done by investors, namely fundamental analysis by analyzing the company's financial statements and technical analysis by analyzing based on stock price movements in the stock market. Several models have been developed to analyze stock price movements to suit the state of the stock market, one of which uses the quantum model (Menezes & Oliveira, 2015; Rosadi, 2017; Murwaningtyas et al., 2019).

Quantum mechanics is a theory that describes the micro world. If the stock index can be considered an object on a macro scale, then every stock on the index is an object on a micro-scale. Stocks are always traded at a specific price, so they are particles. Meanwhile, fluctuating stock prices in the stock market are like waves. Due to the dualistic nature of waves and particles, stocks as objects at the micro-scale can be considered part of a quantum system. This study analyzed the distribution of the return rate of stocks based on the quantum model using the Schrodinger equation (Baaquie, 2007, 2009, 2018).

A stock market is a platform where public shares are traded publicly. Generally, two types of stock markets can be found in various countries. Firstly, the primary market is where the initial offering of shares by the issuer takes place during a specified period determined by the issuer before the shares are traded on the secondary market. In the primary market, the stock price is determined by the underwriter and the company going public. Secondly, the secondary market is where shares previously issued in the primary market are traded long-term.

Stock market growth and economic growth have a relationship with each other. The stock market provides an overview of the economic strength of a country. Countries with good performance in stock markets generally have good economic performance too (Suresh, 2013; Yousif & Elfaki, 2017; Winarno et al., 2021).

Stocks are securities that represent ownership in a company, serving as evidence of individual or institutional capital participation. By owning shares, investors hold a stake in the company's income and assets. They also have the right to attend the General Meeting of Shareholders. Shareholders receive profits in the form of dividends and may also incur capital losses. Dividends are the distribution of profits generated by the company to its shareholders. On the other hand, capital gains refer to profits from the difference between the purchase and selling prices in the secondary market. The stock price represents the value of a share at a specific time and is determined by the forces of supply and demand in the stock market. Fluctuations in stock prices occur due to new information that affects market sentiments. It is common to compare the current stock price with the previous year's price for analysis and evaluation. The stock price at any given time will depend on the cash flows that the average investor expects to receive if the investor buys the stock (Zhang & Huang, 2010; Masoud, 2013).

The profit obtained from the investment is in the form of a return. Returns can be divided into realized returns and expected returns that have not yet occurred but are expected to occur in the future. The rate of return is a measurement of the level of acquisition in investment which is usually expressed as a percentage of the value of the investment. For investments made in the form of shares, investors get a return consisting of dividends and capital gains or a capital loss. The definition of stock price return using the logarithmic function is as follows:

$$r(t) = \log_{10} \frac{p(t)}{p(t-1)} \quad (1)$$

$p(t)$ is the time-dependent stock price (Farida Agustini et al., 2018).

This study analyzed the density distribution of the rate of stock returns using the Schrödinger equation model, which was applied to the distribution of the rate of stock returns of PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. The purpose of this study is to model the distribution of the rate of stock returns using the Schrödinger equation model and compare the results of the distribution of the rate of stock returns between PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. The research aims to demonstrate the benefits of this approach by obtaining the density distribution of the stock returns rate and providing stock analysis based on the quantum model.

2. METHOD

2.1 Time-independent Schrodinger equation with a delta function

Quantum mechanics is the branch of physics that describes the behavior of matter and its interaction with energy at the scale of atoms and subatomic particles. In quantum mechanics, the behavior of a particle can be represented in the form of a wave function obtained from solving the Schrodinger equation. In classical mechanics, a time-independent one-dimensional potential can give

two different motions (Kuru & Negro, 2008). The bound state, in which the probability of finding a particle tends to go to zero beyond a finite distance, and the scattering state, when the particle arrives at the infinity point, then the particle will either slow down or accelerate under the influence of the potential and return to infinity. There are two kinds of solutions to the time-independent Schrodinger equation according to the bound state and the scattered state:

$$\begin{cases} E < V(-\infty) \text{ and } V(+\infty), \text{ bound state} \\ E > V(-\infty) \text{ and } V(+\infty), \text{ dissipated state} \end{cases} \quad (2)$$

All potential will be exhausted at infinity so expressed by:

$$\begin{cases} E < V, \text{ bound state} \\ E > V, \text{ dissipated state} \end{cases} \quad (3)$$

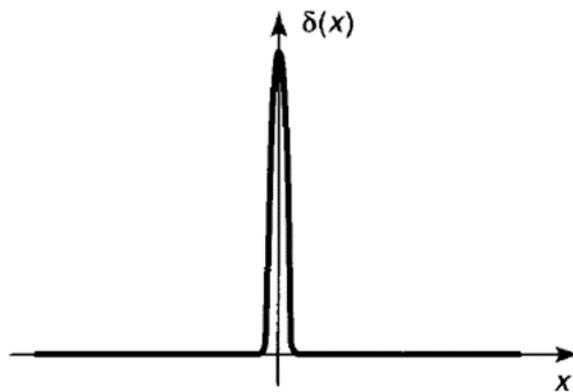


Figure 1. Dirac delta function.

The Dirac delta function (Figure 1) is a function defined according to the equation:

$$\Delta(x - b) = \begin{cases} 0, & \text{for } x \neq b \\ \infty, & \text{for } x = b \end{cases} \quad (4)$$

The time-independent Schrodinger equation for the wave function $\psi(x)$ of a particle at a one-dimensional potential $V(x)$:

$$i\hbar \frac{\partial \psi}{\partial t} + V(x)\psi(x) = E\psi(x) \quad (5)$$

where \hbar is Planck's constant and E is the energy of the particle with delta potential:

$$V(x) = -\lambda\delta(x - b) \quad (6)$$

If λ is negative, it is called delta well potential, while if λ is positive, it is called delta barrier potential. So, the Schrodinger equation of the wave function is stated as follows:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - \lambda\delta(x - b)\psi \quad (7)$$

\hbar is Planck's constant (Griffiths & Schroeter, 2018).

2.2 Data analysis using quantum mechanics

Data analysis begins with knowing the importance of getting the density distribution of stock returns in investing. The normal distribution is more often used for investors to analyze stocks to be invested. It is necessary to assume that a stock is a particle in the potential function of a delta well to know its position to get the density distribution of stock returns using a quantum mechanical model. It

is assumed that $\psi(x)e^{-Et}$ is a solution to the Schrodinger equation such as Equation (7), while E represents the Hamiltonian value of market conditions and t , is time, so the Schrodinger equation becomes:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - \lambda \delta(x-b) \psi = E \quad (8)$$

with

$$\psi(x) = \begin{cases} \sqrt{k} \exp(kx) , \\ \sqrt{k} \exp(-kx) , \end{cases} \quad (9)$$

where $k = \frac{\sqrt{-2mE}}{\hbar}$ and the wave function obtained by $E = \frac{m\lambda^2}{2\hbar^2}$ and $k = \frac{m\lambda}{\hbar^2}$,

$$\psi(x) = \frac{\sqrt{m\lambda}}{\hbar} \exp\left(-\frac{m\lambda}{\hbar^2} |x - b|\right), \text{ for } x \in R \quad (10)$$

and the probability density function of stock price returns is:

$$|\psi(x)|^2 = \frac{m\lambda}{\hbar^2} \exp\left(-2\frac{m\lambda}{\hbar^2} |x - b|\right), \text{ for } x \in R, \quad (11)$$

from Equation (11) the equation for the density probability function of the distribution of stock price returns is:

$$|\psi(x)|^2 = k \exp(-2k|x - b|), \text{ for } x \in R. \quad (12)$$

This study analyzes stock price returns from stock price data. The stock price data used is the stock price data of PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. during 2013-2018. Stock price return data is used as an input parameter in Equation (12) which is processed analytically using Matlab 2012 and Microsoft Excel 2013. The stock price return data are also processed to obtain a stock price return density distribution using a normal distribution processed using Matlab 2012. Results calculation of these data are used as material for analysis. This study obtained the distribution curve of the stock price return density of the two companies using a quantum mechanical model and a normal distribution. Then the results were compared.

3. RESULTS AND DISCUSSION

Stock price return is one factor that motivates investors to invest in the stock market because it can clearly describe changes in the price of their investment results. The stock price return is calculated based on comparing the stock price closed that day to the previous day's stock price. Investors can anticipate the distribution of stock price returns in the event of stock price instability which is feared to experience a very significant decline in stock prices that can harm investors. By knowing the return distribution of a stock, investors can determine the stocks they want to include in their portfolio. The normal distribution or logarithmic distribution is often used to analyze the return of a stock. However, there is a big difference between the normal and the actual distribution. The difference between the two occurs in the actual distribution with a larger kurtosis value.

This study assumes that stock in the stock market is a particle in the potential function of delta wells. The Schrodinger equation can describe stock price returns as an exponential distribution. Classical mechanics assumes that an object is only in one place at a time, so stock price returns based on classical mechanics are assumed to have only one return value. In quantum mechanics, stock price returns are an object, such as an electron or a proton, which has a different position probability, but that position can be measured. The result of the measurement by the market is in the form of stock price returns. In other words, stock price returns can be assumed to be a wave package that can determine the probability of stock price returns.

Figure 2 shows the density comparison of PT. Unilever Indonesia Tbk. stock prices and PT. United Tractors Tbk. using the opportunity density function. The density function of the normal distribution is (Farida Agustini et al., 2018):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (13)$$

where μ is the average stock price, σ is the standard deviation, and π is $22/7$. Equation (13) is used to calculate the standard deviation of the normal distribution of stock prices. The standard deviation of the normal distribution for the stock price of PT. Unilever Indonesia Tbk. from 2013-2018 obtained a value of 0.00746, which is smaller than the share price of PT. United Tractors, which is equal to 0.00918. The greater the standard deviation value, the greater the risk of loss that the stock will obtain. This shows the risk of loss that will be obtained when investing in shares of PT. United Tractors Tbk. is more extensive than PT. Unilever Indonesia Tbk. The smaller standard deviation allows the resulting density function to be larger as shown in Figure 2. Distribution of stock prices of PT. Unilever Indonesia Tbk. shows a higher peak point and a denser distribution width. This shows the stock price return of PT. Unilever Indonesia tends to be more stable so that the stock price fluctuations they experience are not too extreme compared to the stock price returns of PT. United Tractors Tbk.

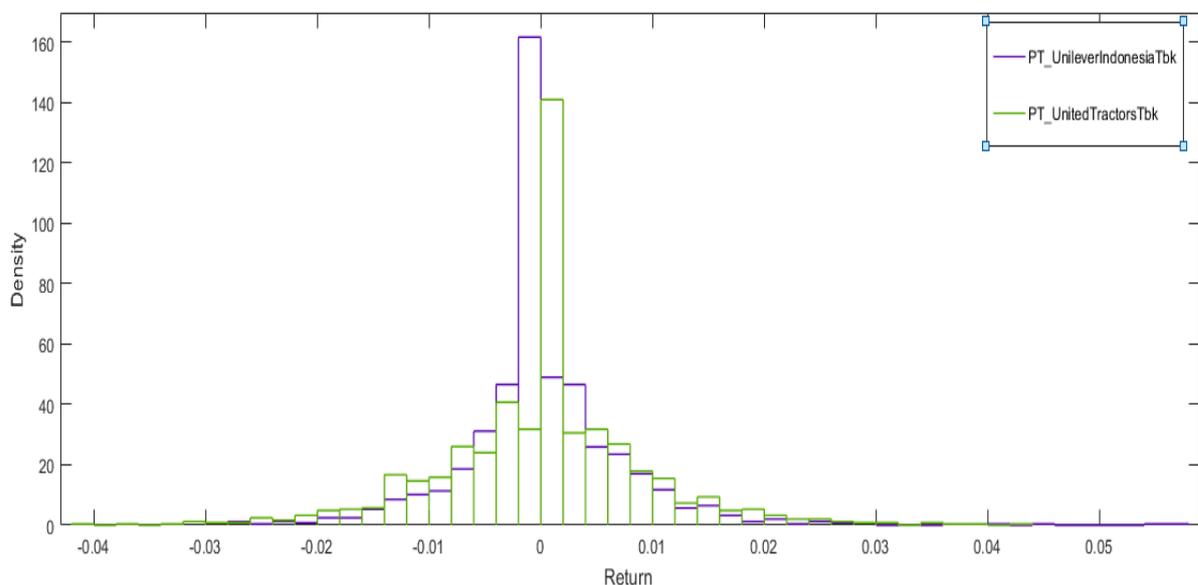


Figure 2 The comparison curve of the Probability Density Function of stock returns from PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. The calculation results of the Probability Density Function curve are calculated using Equation (13).

Based on the density function calculation from the quantum mechanics model, the density distribution curve is obtained for the stock price return of PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. showed in Figures 3 and 4. The density distribution curves of the two stock prices using the Probability Density Function and the distribution density function from the quantum mechanics model show the same shape as the Dirac delta function curve. Dirac's delta function $\delta(x - b)$ is zero when x is not equal to b , while Dirac's delta function goes to infinity $(x - b) = 0$, this means $x = b$. The delta potential is the potential derived from the Dirac delta function $\delta(x)$. This potential will be zero at all points except one. Stock price return density from PT. Unilever Indonesia Tbk. and PT. United Tractors Tbk. shows a higher density when the return value gets closer to the return value equal to zero. This shows that the stock return generated at any time is close to parameter b , as in Equation (12). The stock density distribution curve of PT. Unilever Indonesia Tbk. using a quantum mechanical model shows a higher peak point than the Probability Density Function. This can occur due to differences in the parameter values used in the distribution density function of the quantum mechanics model with the

Probability Density Function. The input parameters used in the distribution density function of the quantum mechanics model are shown in Tables 1 and 2.

In this study, the normal distribution density function that calculates the actual data is proven to obtain the exact prediction as the calculation using the stock price density distribution from the quantum mechanical model for data from PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk.

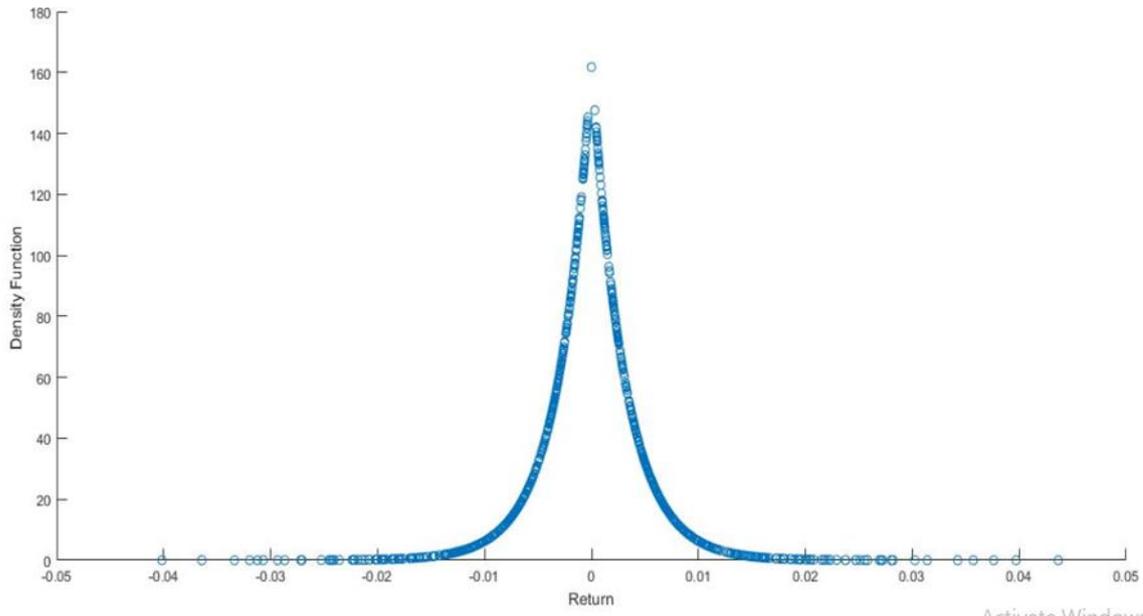


Figure 3 The density probability function curve of the stock price return of PT. United Tractors Tbk. was calculated using the Schrodinger equation model (Equation (12)) from 2013 to 2018.

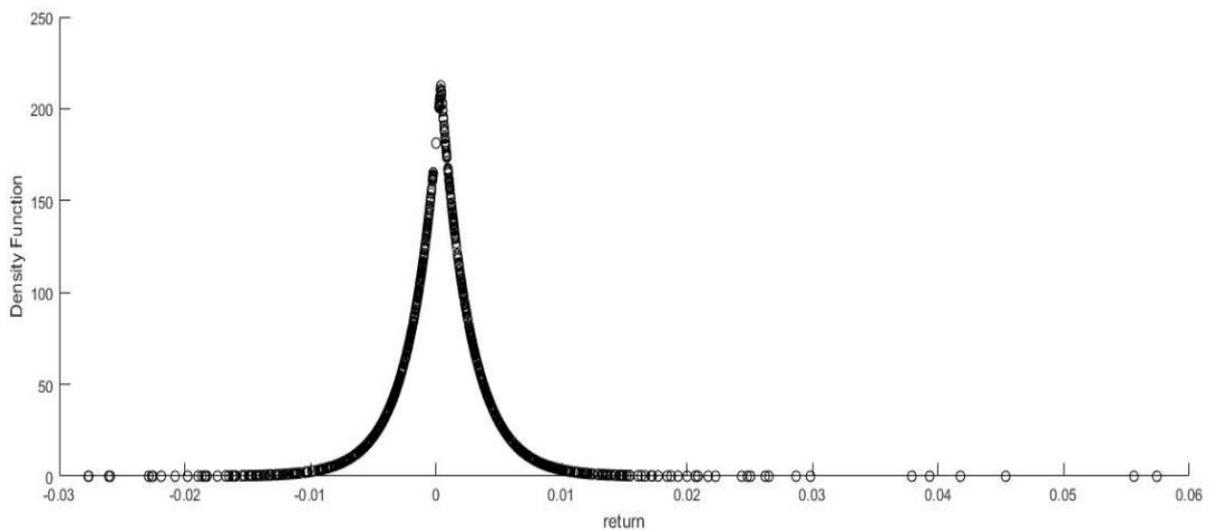


Figure 4 The density probability function of the stock price return of PT. Unilever Indonesia Tbk. was calculated using the Schrodinger equation model (Equation (12)) from 2013 to 2018.

There are three parameters, namely λ , m , and b in the quantum mechanics model, as shown in Equation (11), while from Equation (12) the probability for stock price returns is b , and the variation of stock price returns is $h^2/m\lambda$. When the parameter m is getting bigger, the change of stock price return gets smaller. The parameter λ is the uniformity of market behavior. If investors have the same view of the stock market, then is more significant and the variation of the stock price return is smaller. Parameter

b is the average of stock price returns. The distribution density function of the quantum mechanics model for stock price returns is influenced by several parameters listed in Tables 1 and 2. Parameters b and k contained in Equation (12) are obtained from:

$$b = \frac{1}{N} \sum_{i=1}^N X_i \tag{14}$$

and

$$k = \frac{N}{\sum_{i=1}^N |X_i - b|} \tag{15}$$

where X is the stock price return each month. In this study, there is a total data, $N=1234$, from stock prices obtained for five years which is used for data analysis. If T is $1/2k$ in Equation (12) then the density function of the stock price return distribution becomes:

$$|\psi(x)|^2 = \frac{1}{2T} \exp\left(-\frac{|x-b|}{T}\right) \tag{16}$$

Parameter b can be expressed as the profitability of the stock price because it is the average return of the stock price. The larger the parameter b , usually the more attractive the investment. A negative value in parameter b can indicate that the stock tends to experience risk in the form of capital loss due to its negative average return.

Based on Tables 1 and 2, the value of parameter b from PT. Unilever Indonesia Tbk. is bigger than PT. United Tractors Tbk. tends to be more stable because the average return is more positive than PT. United Tractors Tbk., so it can be concluded that the shares of PT. Unilever Indonesia Tbk. is more attractive for investors to invest. The parameter T can be assumed to be the temperature of the stock, so a value of $1/k$ can define whether the stock is hot or cold. The standard deviation of the stock price return X is equal to $(2T)^{1/2}$, which imposes a risk on the stock based on Markowitz's theory. The greater the standard deviation, the greater the potential risk obtained. The standard deviation of the share price of PT. Unilever Indonesia Tbk. from 2013 to 2017 was 0.0685 while the share price of PT. United Tractors Tbk. amounted to 0.0784. Therefore, stock risk can be considered by analyzing the temperature of the stock.

The K values in Tables 1 and 2 are kurtosis values. The kurtosis value is the degree of peak height or sharpness of a distribution. The bigger the kurtosis, the better the data will be. The estimation of kurtosis for stock price returns is obtained based on Equation (12) as follows (Ruppert, 2004; Markowitz, 1959; Markowitz, 1959):

$$K = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 \tag{17}$$

The kurtosis value indicates whether a distribution is normally distributed or not. Based on Tables 1 and 2, the value of kurtosis with different time spans for PT. Unilever Indonesia Tbk. and PT. United Tractors shows a different value. The kurtosis value for the two stocks has a value of more than three, which is the kurtosis value for the normal distribution, indicating that the curve is more pointed than the normal distribution. A higher kurtosis value can indicate that investors will experience extreme returns on stock returns, both positive and negative returns. Based on the calculation with Equation (17) the kurtosis value of PT. Unilever Indonesia Tbk. is bigger than PT. United Tractors Tbk. The more data used, the better the analysis of a stock will be. This can describe how the state of stock in the long term.

Table 1. The calculation results of the parameters on the Schrodinger equation model for the stock price data of PT. United Tractors Tbk. from 2013 to 2018 using Equations (14), (15), and (17).

Time (Year)	B	k	K	T
2013	0.000321	145.367	6.0359	0.00344
2013-2014	2.29×10^{-5}	210.237	8.3849	0.00238
2013-2015	0.000149	174.879	6.6245	0.00286
2013-2016	9.17×10^{-6}	165.384	5.8734	0.00302
2013-2018	1.62×10^{-5}	162.635	5.4732	0.00307

Table 2. The calculation results of the parameters on the Schrodinger equation model for the stock price data of PT. Unilever Indonesia Tbk. from 2013 to 2018 using Equations (14), (15), and (17).

Time (Year)	B	k	K	T
2013	0.00094	40.8673	11.9196	0.00359
2013-2014	0.00067	206.463	13.9515	0.00242
2013-2015	0.00043	189.839	11.0039	0.00263
2013-2016	0.00034	198.319	11.0503	0.00252
2013-2018	0.00038	213.0265	8.933	0.00235

4. CONCLUSION

Investors need the density distribution of a stock price to monitor the phenomenon of stock price instability because investors have concerns that they will experience a decline in stock prices which will cause very significant losses. Modeling the distribution of the rate of return of stock prices using a quantum mechanical model by assuming that a stock price is a particle that is in a potential function of a delta well so that a particle analogous to the return of a stock price can be identified. This study calculates the density distribution of stock prices using a quantum mechanical model for data from PT. United Tractors Tbk. and PT. Unilever Indonesia Tbk. This study shows the stock price of PT. Unilever Indonesia Tbk. is more attractive for investors to invest with a more stable average return on stock prices with a smaller risk of loss than the stock price of PT. United Tractors Tbk. Therefore, the stock price density distribution using the quantum mechanical model has a kurtosis value above 3 which is higher than the kurtosis value in the normal distribution.

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