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An Exact Solution of Nonlinear Schrödinger Equation in a Lossy Fiber System Using Direct Solution Method

Zulfi Abdullah[1*], Trengginas Eka Putra Sutantyo[1], Mahdhivan Syafwan[2], Ahmad Ripai[1], Hanifah Azzaura Musyayyadah[3], Mohamad Nazri Abdul Halif[3]

[1]Theoretical Physics Laboratory, Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Andalas, Kampus Unand Limau Manis, Padang 25163, Indonesia
[2]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Andalas, Kampus Unand Limau Manis, Padang 25163, Indonesia
[3]Institute of Engineering Mathematics, Universiti Malaysia Perlis (UNIMAP) Pauh Putra Campus, 02600, Arau, Perlis, Malaysia

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Corresponding Author:

Zulfi Abdullah
Email: zulfi@sci.unand.ac.id

ABSTRACT

We present an exact solution of the nonlinear Schrödinger equation (NLSE) for beam propagation in nonlinear fiber optics. It is a lossy fiber system with the beam as solitons. Fiber losses are understood to reduce the peak power of solitons along the fiber length. That is due to its value depending on the fiber attenuation constant of α. Considering fiber loss features on the equation, we write one set modification of the NLSE and make models the main topic of our work. We solved the model and found a straightforward analytical solution of modified NLSE for the system via the direct solution method. To the best of our knowledge, no literature has presented such as solution yet. By substituting them into equations, we validate solutions. It is valid as an exact solution to the NLSE. Lastly, we found a solution offering soliton propagation suitable for the system under study.

1. INTRODUCTION

The nonlinear Schrödinger equation (NLSE) is a classical field equation used in physics to study light propagation in nonlinear fiber optics and various other phenomena such as Airy beams, rogue waves, plasmas, Bose-Einstein condensates, superconductivity, quantum mechanics, and nano-optical fibers (Zhang et al., 2019; Nisar et al., 2022). NLSE is a crucial theoretical model in physics owing to its unique mathematical features, which have piqued the interest of numerous researchers. NLSE-like equations have recently been applied to biophysics, specifically in DNA dynamics systems (Sutantyo et al., 2022). Additionally, nonlinear saturation in photorefractive media solitons research has captured interest (Katti, 2019; Ripai, Abdullah, et al., 2021; Ripai, Sutantyo, et al., 2021).

The mathematical complexity of NLSE leads to challenges in finding solutions for many courses. Various settlement methods were developed using mathematics and computational techniques.

Researchers have studied the NLSE using multiple methodologies in the context of beam propagation in nonlinear fiber optics. The NLSE is crucial for the study of long-distance optical communications and all-optical super-fast switching devices (Nisar et al., 2022). Zhou et al. (2016) also applied the (G'/G) expansion method to solve the problem related to NLSE in fibers. Inc et al. (2017) employed the GPRE method with sub-ODE Bernoulli and sub-ODE Riccati-Bernoulli. Yin et al. (2018) provided their solutions using the fourth-order split-step Runge-Kutta method. Bakodah et al. (2019) improved the adomian decomposition, a semi-analytical method for solving nonlinear differential equations, in their work. Ripai et al. (2020) successfully confirmed that the numerical split-step Fourier method is a reliable benchmark for the related case. Subsequently, Kudryashov (2021) employed the Jacobi elliptic sine method, while Nisar et al. (2022) employed the two analytical methods of the sub-ODE Bernoulli and the (G'/G) expansion. In contrast, Liu et al. (2022) conducted numerical research on the NLSE of fiber optics using the split-time-step Crank–Nicolson method. Later, Saputra et al. (2022) verified the bilinear formula for the NLSE problem.

This paper presents a concise mathematical model that accurately solves the NLSE in nonlinear fiber optics. The fiber optic is treated as a lossy system, following Böhm and Mitschke's (2007) description, and is further simplified by Wang et al. (2021). Loss within the fiber arises from the absorption of optical beam power, or energy, due to its attenuation constant, α, as described by Böhm and Mitschke (2007) and Wang et al. (2021). When studying beam propagation within a system, the attenuation feature is integrated into the NLSE. Agrawal (2013) demonstrates that we can derive this integration through beam propagation methods, resulting in a set of modified NLSE. This is commonly referred to as the model of perturbative NLSE. It's important to note that perturbation is lossy, as attenuation reduces the signal over distance. Certain methods outlined in the previous paragraph may present difficulties and require extensive mathematical development. Thus, we offer the direct solution method (refer to section 2) as an alternative to a simpler technique that avoids complex calculations. This method involves modifying the NLSE for a lossy fiber system and solving the problem by creating an ansatz definition beforehand, which is a mathematical model of assumptions related to the equation's solution function. After substituting the ansatz into the equation, we are able to reduce the NLSE to an ordinary differential equation (ODE).

To find the analytical solution of a soliton (specifically, a bright soliton), we complete the solitary wave boundary condition of the ODE by relying on a simple calculus technique—an integral involving an inverse hyperbolic function. Additionally, in section 2, we discuss the properties of bright solitons in lossy fibers using the obtained solution. Meanwhile, in Section 3, we validate the bright soliton solution we discovered as an exact NLSE solution for optical beam propagation in nonlinear fiber optics. Finally, in Section 4, we present our conclusions.

2. NLSE MODEL OF LOSSY FIBER SYSTEMS AND DIRECT SOLUTION METHOD

In this paper, we mainly consider the NLSE unit in lossy fiber systems as following equation (Böhm & Mitschke, 2007; Wang et al., 2021):

$$i\frac{\partial q}{\partial \xi}+\frac{1}{2}\frac{\partial^2 q}{\partial \tau^2}+i\frac{\Gamma}{2}q+|q|^2 q=0. \tag{1}$$

$\Gamma = \alpha L_D$ is the quantity related to fiber losses ($\Gamma<1$), α is the attenuation constant, and L_D is the fiber optics dispersion length. Besides, $\xi = z/L_D$ is the dimensionless form of the pulse propagation distance, and $\tau = T/T_0$ is the time unit, with T_0 being the width of the incident pulse. We know that $|q|^2q$ is the fiber optic nonlinearity, i.e., nonlinear Kerr, which balances the dispersion effect or attenuation. Since the third term is related to fiber losses, it reduces the peak power of solitons along the fiber length if we consider them as solutions. Here, we consider soliton as the object of our studies. When the second term in (1) is positive, we work in an anomalous dispersion system where the soliton solution is a bright soliton (Agrawal, 2013; Saputra et al., 2022). Now, let us solve Equation (1) using direct solution methods.

First, let us consider ansatz or the solution function of Equation (1) to satisfy:

$$q = g(\tau)\exp(i\nu\xi). \tag{2}$$

$g(\tau)$ is the envelope soliton function, whose value will be determined. ν is the shift of the nonlinear propagation constant, whose value will also be determined. Substitute Equation (2) into Equation (1) to determine both values. It leads us to an ODE in Equation (3) as follow:

$$\frac{1}{2}\frac{\partial^2 g(\tau)}{\partial \tau^2} = \nu\, g(\tau) - i\frac{\Gamma}{2} g(\tau) - g(\tau)^3. \tag{3}$$

In this situation, Equation (3) becomes the dynamical evolution model for the beam envelope in fibers. Again, we consider it as a soliton beam. So that Equation (3) means a soliton, specifically bright solitons, $g(\tau)$ must be a normalized function: $0 \le g(\tau) \le 1$, satisfy the boundary conditions: $g(0) = 1$, $\dot{g}(0) = 0$, $g(\tau \to \pm\infty) = 0$, and all derivatives $g(\tau)$ for $\tau \to \pm\infty$ equal to zero. Integrating Equation (3) and completing the boundary conditions, we get:

$$\frac{1}{2}\left(\frac{\partial g}{\partial \tau}\right)^2 = \left(\nu - i\frac{\Gamma}{2}\right) g^2 - \frac{1}{2} g^4, \tag{4}$$

with

$$\nu = \frac{1}{2}(1 + i\Gamma). \tag{5}$$

Equation (4) based on Equation (5) can be evaluated quickly by simple calculus techniques, i.e., an integral involving the inverse of hyperbolic functions. We found $g = \text{sech}(\tau)$ to be the value of the envelope soliton functions. Thus, the NLSE solution (Equation (1)) becomes intact in the form:

$$q = \text{sech}(\tau)\exp\left(\frac{1}{2}(i - \Gamma)\xi\right). \tag{6}$$

Equation (6) is the analytical solution of Equation (1) that we found in the fiber optics context as lossy fiber systems. Here, Equation (6) offers bright soliton profiles (denoted by the secant-hyperbolic function), which decay exponentially by the fiber losses Γ (Figure 1(b)). It is because fiber losses reduce the peak power of solitons along the fiber length. In other words, the fundamental soliton width increases during propagation due to lossy power (Figure 1(a)).

Figure 1 represents the evolutionary curve of bright soliton in the system based on the NLSE analytical solution of Equation (1) in Equation (6). Refering to the curve, we can understand that the bright soliton profile that the secant-hyperbolic function offers in Equation (6) evolves along the fibers. Here, we set their evolution for a very long propagation distance with the value of the associated quantity of fiber losses ($\Gamma = \alpha L_D$) varying between 0 to 1. Evolution is characterized by a soliton width curve (T/T_0) that increases dramatically along the fibers in Figure 1(a), and the peak intensity curve ($|q|^2$) decreases in Figure 1(b). When the value of fiber losses Γ corresponding to the attenuation constant of the α set increases, as information in the figures, we find that the change in the width of the bright soliton (T/T_0) in the unit of distance (z/L_D) becomes more significant (Figure 1(a)). That indicates that the higher fiber losses of the system make for a more drastic reduction in power or soliton energy. As a result, the bright soliton profile will be found to disappear faster. In other words, it does not have invariant propagation along the fibers. It did not meet the experimentalists' expectations (Agrawal, 2011).

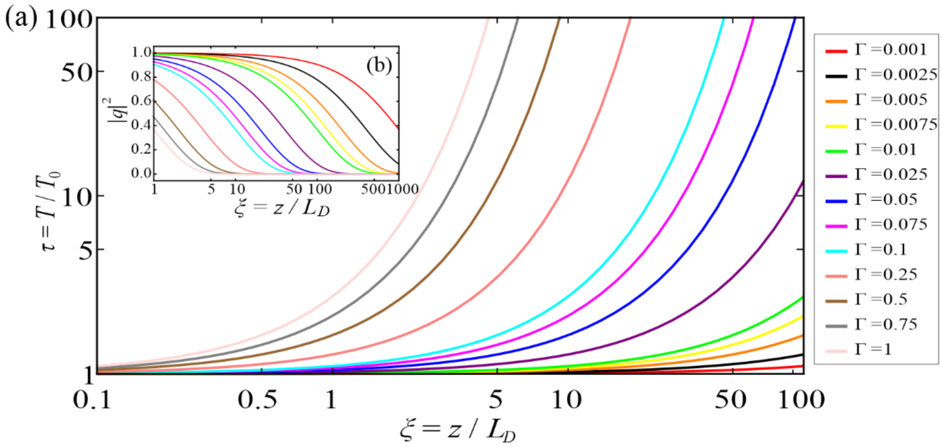

Figure 1 Bright soliton evolution in lossy fiber system based on solution (6): (a) the soliton width curve versus distance unit of ξ and (b) the soliton peak evolution.

The silica fibers we use today are commonly created by a modified chemical vapor deposition (MCVD) process and exhibit a minimum α_{dB} loss of about 0.2 *dB*/*km* near 1.55 μm of light wavelength (λ). We again calculated the loss value through the $\alpha_{dB} \approx \lambda^4$ relationship and illustrated it in Figure 2 (look at the solid black line). Following the unit conversion rules in Agrawal (2013), this value corresponds to the attenuation constant $\alpha \approx 0.04$/km. From the minimum α_{dB} loss itself, we can calculate the fiber losses quantity Γ for the width of the incident pulse T_0 = 4 *ps* and the dispersion parameter β_2 = -20 ps^2/km (experimental standard values) reach:

$$\Gamma = \alpha L_D = \alpha \frac{T_0^2}{|\beta_2|} \approx 0.04\!\!\left/\!{km}\right. \frac{(4\,ps)^2}{\left|-20\ {ps^2}\!\!\left/\!{km}\right.\right|} \approx 0.04\!\!\left/\!\cancel{km}\right. \frac{16\ \cancel{ps^2}\ \cancel{km}}{\left|-20\ \cancel{ps^2}\right|} \approx 0.032. \tag{7}$$

It has also shown a fairly dramatic change in the width of the soliton. Notice the blue curves in Figure 2, they are right between the black and yellow curves in Figure 1. Despite all the experimental importance, we assess that solution in Equation (6) is quite accurate in predicting the evolution of soliton in a lossy fiber system. While the stability of the realistic soliton needs to be a further concern.

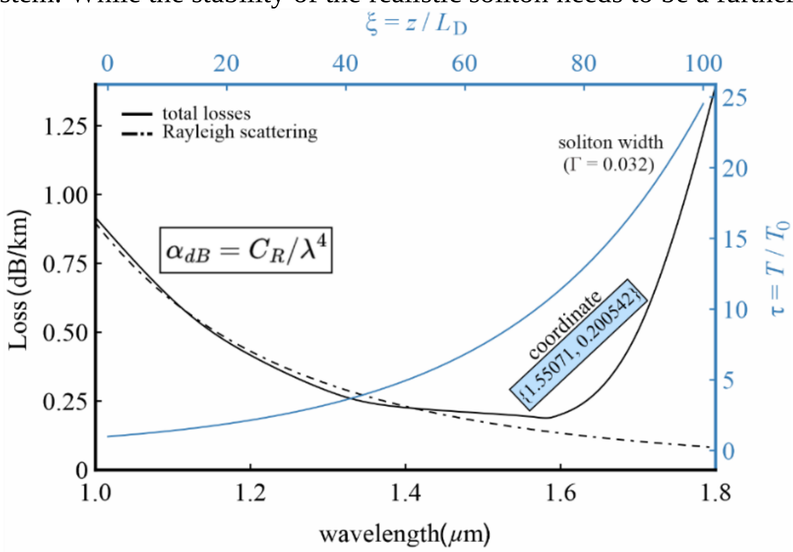

Figure 2 Loss spectrum versus wavelength of a single-mode silica fiber (solid black line) and the contribution from Rayleigh scattering (dot-dashed curve). An blue curve illustrates the soliton width at minimum losses (α_{dB} = 0.2 *dB*/*km*) near 1.55 μm.

Let's go back to Figure 1, we also show the evolution peak of the bright soliton, which is in Figure 1(b). They decay as the quantity of fiber losses Γ increases. It is not strange if we are in the realm of nonlinear physics. The decay of its peak intensity always follows the dilation of the soliton pulse profile. We have compared the two, their variation is so consistent for the same quantity of fiber losses Γ (Figures 1(a) and 1(b)). Since the width of the bright soliton profile dilated, the peak intensity decayed. The problem becomes more significant when the quantity related to fiber losses Γ is higher (Figure 1). Someone may ask how the dilating and decay of bright soliton peaks occur. That's because of the nonlinearity and dispersion effect of fibers (Agrawal, 2011). At the beginning of this section, we alluded to the fibers' nonlinearity in the form of nonlinear Kerr denoted by the third term in the NLSE in Equation (1). Nonlinearity causes the width of the bright soliton pulse profile to narrow and the peak of its intensity to rise along the fibers. In contrast to dispersion: characterized by a second term of Equation (1), it gives rise to the dilation effect on the pulse profile followed by the decay of the peak intensity. We recommend the books by Agrawal (2013) and Kivshar & Agrawal (2003) for readers to more easily understand this issue. For the bright soliton to have established stability, nonlinearity and dispersions must be at the right balance, and we do not find them in realistic cases (lossy fiber system). Again, we assess that solution in Equation (6) quite correctly predicts the evolution of solitons in lossy fibers.

Next, we also compare the evolution of bright solitons in ideal (lossless) and lossy (realistic) fiber systems. It is clear by taking the quantity of fiber losses $\Gamma = 0$ (ideal) and $0 < \Gamma < 1$ (realistic) for the Equation (6). Now let's consider Figure 3, when fiber losses Γ zero-valued system, bright solitons are shown not to evolve or stabilize along the fibers. This ideal case meets the experimentalist' expectations (Agrawal, 2011; Mollenauer & Gordon, 2006). Since the Γ parameter is zero, Equation (6) is reduced to:

$$q = \operatorname{sech}(\tau)\exp\left(i\,\xi/2\right). \tag{8}$$

We know this as the exact solution of NLSE in ideal fibers (lossless) where the solitons offered are fundamental (Agrawal, 2011; Ripai et al., 2020). For the parameter $0 < \Gamma < 1$, bright solitons then evolved along the fibers. Notice the plus-marked red lines in Figure 3 (Γ=0.05), solitons peak intensity decay as we found above (Figure 1(b)). From this, we can understand that solutions (6) are so powerful in describing the evolution of soliton fibers from Equation (1), where they can be an ideal case for the context of lossless fibers, i.e., when we choose $\Gamma = 0$ for our solutions. Lastly, we felt the need to validate the solution in Equation (6) and it was presented in Section 3. Before that, we were first interested in presenting the peak intensity data in Figure 3 to support a qualitative perspective. Here, we present Table 1 as their quantitative point of view.

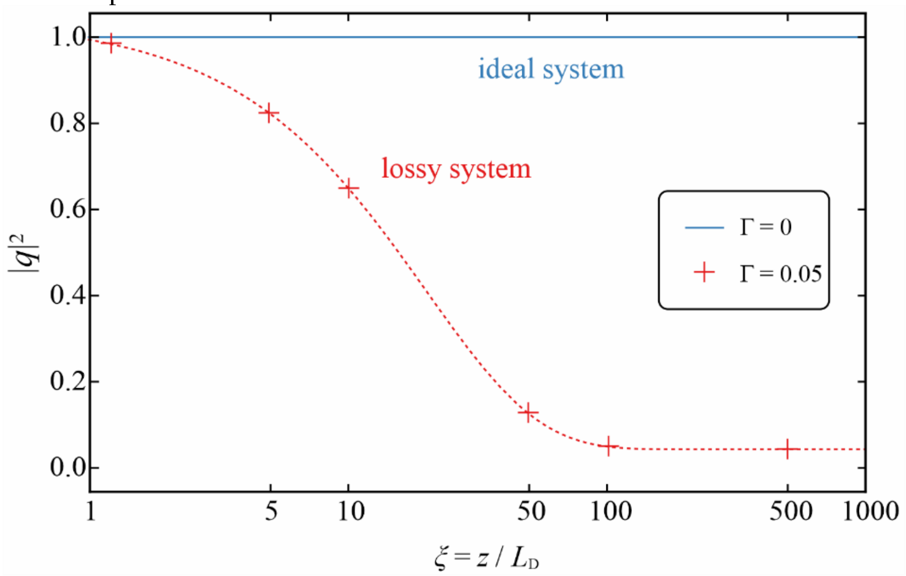

Figure 3 Peak evolution of bright soliton in ideal (blue lines) and lossy fiber system (plus-marked red lines).

Table 1 is the peak intensity data of bright soliton along the fiber when Γ = 0 (ideal system) and 0 < Γ < 1 (lossy system); we take the value Γ = 0.05. Pay attention to the tables, they are completely stable for the ideal system and evolved for the lossy system. The peak intensity data of bright soliton above confirmed they experienced peak intensity decay along the fiber as a harbinger of evolution. That is as presented in Figures 1-3. Thus, the solution in Equation (6) we found was quite appropriate and so powerful in describing the evolution of bright soliton in a lossy fiber system. For validation, let's prove the exact solution in section 3 below.

Table 1 the peak intensity of the bright soliton in ideal (Γ = 0) and lossy fiber system (Γ = 0.05).

Number	**Propagation Distance** $\xi = z/L_D$	**Peak Intensity of the Ideal System** $\lvert q\rvert^2$	**Peak Intensity of the Lossy System** $\lvert q\rvert^2$
1	0	1.000000000	1.000000000
2	150	1.000000000	0.000553084
3	250	1.000000000	3.72665×10^{-6}
4	350	1.000000000	0.02511×10^{-6}
5	450	1.000000000	0.16919×10^{-9}
6	550	1.000000000	0.11399×10^{-11}
7	650	1.000000000	0.76812×10^{-14}
8	750	1.000000000	5.17556×10^{-17}
9	850	1.000000000	3.48726×10^{-19}
10	950	1.000000000	0.23497×10^{-20}
11	960	1.000000000	1.42516×10^{-21}
12	970	1.000000000	8.64406×10^{-22}
13	980	1.000000000	5.24289×10^{-22}
14	990	1.000000000	3.17997×10^{-22}
15	1000	1.000000000	1.92875×10^{-22}

3. VALIDATION OF THE EXACT SOLUTION

For solution in Equation (6) to be accepted and valid, we validate the solution as an exact solution of NLSE unit (Equation (1)). It is clear by substituting Equation (6) into Equation (1). In substitution process, the first term of differential $\partial q/\partial\xi$ becomes:

$$\frac{\partial q}{\partial \xi} = \frac{\partial}{\partial \xi}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right) = \frac{1}{2}(i-\Gamma)\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right). \tag{9}$$

While the second term $\partial^2 q/\partial\tau^2$ provides value:

$$\begin{aligned}\frac{\partial^2 q}{\partial \tau^2} &= \frac{\partial}{\partial \tau}\left\{\frac{\partial}{\partial \tau}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)\right\} \\ &= \operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right) - 2\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right).\end{aligned} \tag{10}$$

For the $|q|^2$ component, we get:

$$|q|^2 = q^* \cdot\ q = \operatorname{sech}^2(\tau). \tag{11}$$

Finally, we can arrange the substitution process Equation (6) into the NLSE unit in Equation (1). It gives the same value for both sides of the equation, i.e., equal to zero (exact).

$$i\frac{\partial q}{\partial \xi}+\frac{1}{2}\frac{\partial^2 q}{\partial \tau^2}+i\frac{\Gamma}{2}q+|q|^2 q=0.$$

$$\text{step1}\quad i\frac{1}{2}(i-\Gamma)\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+\frac{1}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)-\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)$$
$$+i\frac{\Gamma}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)=0$$

$$\text{step 2}\quad -\frac{1}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)-i\frac{1}{2}\Gamma\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+\frac{1}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)$$
$$-\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+i\frac{\Gamma}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)=0$$

$$\text{step 3}\quad -\frac{1}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+\frac{1}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)-i\frac{1}{2}\Gamma\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)$$
$$+i\frac{\Gamma}{2}\operatorname{sech}(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)-\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)+\operatorname{sech}^3(\tau)\exp\left(\frac{1}{2}(i-\Gamma)\xi\right)=0$$

$$\text{step 4}\quad 0=0.\quad \underline{\text{proven}}$$

We have validated that the NLSE analytical solution of Equation (1) that we found in equation (6) via the direct solution method is an exact solution of the equation. As we have described in section 2, the direct solution method is quite simple to extract the soliton-type solution from the NLSE even though the fiber is considered a lossy system (involving the integration of $i\Gamma q/2$ in the equation). It differs from some of the methods we mentioned in the introduction, which often undergo relatively complex mathematical developments. Furthermore, the soliton solution given through the direct solution technique is also relevant. By the context of the lossy system that we discussed, even our solution can be transformed into a lossless NLSE solution, which provides a fundamental soliton solution that is in line with the expectations of the experimentalists (look back at Figure 3). However, we understand that each method has advantages and disadvantages. To the best of our understanding, each technique provides a different solution in the context of nonlinear dynamics (solutions are always technique based). For example, in a context that requires the interaction of two solitons in the system more relevant if it is extracted using a Bilinear formula (Yan & Chen, 2022). However, if there was no interaction (as in our case), we thought the more straightforward methods we use could be much more interesting than the Bilinear method, which is relatively long in mathematics.

4. CONCLUSION

This study has provided a deep dive into the nonlinear Schrödinger equation (NLSE) within the realm of lossy fiber optics, addressing the significant challenge of incorporating fiber losses into the NLSE's framework. Our introduction highlighted the NLSE's broad applicability and the complexities involved in deriving solutions, especially in systems where loss mechanisms play a crucial role. We set out to simplify these solutions, aiming to bridge the gap between theoretical models and practical applications in fiber optics.

In conclusion, our work has successfully executed this task, utilizing the direct solution method to integrate the NLSE for a lossy fiber system. The result is an analytical solution, proven to be exact, that manifests as a bright soliton. This solution is particularly valuable because it captures the essence of soliton behavior under the influence of fiber losses characterized by the parameter Γ, which is intrinsically linked to the attenuation constant α. The solution illustrates the physical reality of solitons in lossy fibers, revealing how their peak powers decay exponentially with distance and how this decay influences the soliton's width along the fiber.

Our findings resonate with the objectives stated in the introduction, providing a more straightforward and exact solution to a complex problem that has implications for long-distance optical communications and other applications in nonlinear fiber optics. This research not only contributes a novel solution to the scientific community but also paves the way for future investigations into the stability and dynamics of solitons in similar lossy systems.

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