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Modeling of Dynamics Object with Non-Holonomic Constraints Based on Maple in Cylinder Coordinate $\mathbb{R} \times S^1 \times SO(3)$

Melly Ariska[1*], Hamdi Akhsan[1], Muhammad Muslim[1], Jesi Pebralia[2]
Arini Rosa Sinensis[3], and Tine Aprianti[4]

[1]Physics Education, Faculty of Teacher Training and Education, Sriwijaya University
Palembang-Prabumulih Street KM 32 Indralaya, Ogan Ilir, South Sumatra 32155, Indonesia
[2]Physics Department, Faculty of Sciences and Technology, Universitas Jambi
Jambi - Muara Bulian Street No.Km. 15, Muaro Jambi 36122, Jambi, Indonesia
[3]Physics Education Program, STKIP Nurul Huda OKU Timur
Kotabaru Street Sukaraja, East OKU 321161, South Sumatra, Indonesia
[4]Chemical Engineering Department of University of Western Australia

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Corresponding Author:

Melly Ariska
Email: mellyariska@fkip.unsri.ac.id

ABSTRACT

Reliable real-time planning for dynamic systems is crucial in today's rapidly growing automated ecosystem, such as the environment and methods of planning a robotic system. This paper describes the rigid dynamics system with non-holonomic constraints on the $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ configuration space. The method used is the motion planning network and numeric treatment using physics computation which can be used for non-holonomic object systems that move in real-time with Jellets Invarian (JI) approach. The JI approach can result in a motion system equation and evaluate the model of an object with non-holonomic constraints and also display experimental results for navigation in the $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ configuration space. The motion system with non-holonomic constraints used is Tippe top (TT). TT is a toy like a top which when rotated will flip itself with its stem. The author have finished in simulating the dynamics of TT motions in real time with the initial states that have been described with various coordinate in the $\mathbb{R}^2 \times \mathrm{SO}(3)$ configuration space. Based on the results of previous studies on similar objects, TT was solved by the Eular-Lagrange Equation, Routhian Reduction Equation and Poincare. The author succeeded in describing the dynamics of TT motion in real time with predetermined initial conditions with various coordinates in the $\mathbb{R}^2 \times \mathrm{SO}(3)$ configuration space.

1. INTRODUCTION

Geometric mechanics is study of physics, mathematics and engineering, which contains many research topics. Many ideas and developments in geometric mechanics have played a role in other scientific disciplines to deal with practical problems (Holm, 2011). Applied geometric mechanics can be found in various fields such as robotics, vehicle dynamics, and locomotive motion in various animal movements involving non-holonomic mechanics (Ariska et al., 2020a). The rigid body phenomenon analyzed in the previous study only observed about rotating, slipping and rolling, with holonomic constraints of objects on a flat surface. However, their research does not analyze numerical solutions for asymmetric bodies with holonomic constraints. The non-holonomic system was introduced in mechanics by Lewis (2017) and Ariska et al. (2020a, 2020b), which meant that the system experienced constraints that limited the speed of the system particles in the configuration space (Holm et al., 2009). Constraints are conditions that limit the motion of a mechanical system so that reducing the degrees of freedom (Ariska et al., 2018). Holonomic constraints always involve the speed of the system and can be written in the form of degree one (Shimomura et al., 2005). These constraints are found in the configuration space and do not reduce the degree of freedom and limit the movement of the system in the configuration space and momentum (Ariska et al., 2020b). The system can be described by diversity (manifold), which is an effort to build the coordinates of a space in the form of a collection of points, lines or functions (Sriyanti et al., 2020). The configuration space used in the form of Lie groups, then to look for constraints group theory can be employed, but in this study we only need to use differential calculations (Ariska et al., 2019; Ciocci et al., 2012; Ciocci & Langerock, 2007; Zobova, 2012).

Tippe top (TT) is one of complex body rigid with non-holonomic constraints. Objects with non-holonomic constrain are governed by dynamical equations which depend on the time-derivative of the system's configuration space (Bou-Rabee et al., 2004; Johnson et al., 2020; Zobova, 2012). These constraints arise in many applications, ranging from mobile robot navigation to needle steering in robot surgery (Zobova, 2011). Generalizing comes in cases where the system's control space is of a lower-dimension than its configuration space. For instance, in car-like robots, the control inputs are the linear and angular velocities, while the configuration or robot motion space is three-dimensional. Consequently, a feasible trajectory in the robot's configuration space might not be feasible with respect to the system's dynamics (Broer et al., 2001; Fokker, 1952). In this paper, the dynamics analyzed are the dynamics of the Tippe Top (TT) moving in the configuration room $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$. TT as an axially symmetric sphere rolling and gliding on a $\mathbb{R} \times \mathrm{S}^1 \times SO(3)$ surface or inner cylinder surface according to Jellets Invarian (JI) reduction method, equations for motion of a rigid body.

The equations of motion are non integrable and are difficult to be analyzed. A TT is a simple example of a locomotive motion system with non holonomic constraints, but the study of mechanics is not trivial. TT is a toy that has the form of a truncated sphere with a small peg. When the toy is spun on its spherical part on a flat surface or tube inner surface it will start to turn upside down to spin on its peg (Rauch-Wojciechowski et al., 2005). We will approach the modeling from the perspective of a controlled Jellets Invarian (JI), because JI is one way to express system energy firmly (Smale, 1970). In addition, Jellets Invarian is a dynamical system that can be described by a set of differential equations. It was first shown by Jellet (1872) by an approximate argument, and later proved Routh (1884) that the system, even if dissipative, has a conserved quantity. In this paper, the TT motion equation is derived through JI method by physics computation based on Maple 18. Systems with non-holonomic constraints can be hidden by establishing or select one of Levi-Civita connections (Bou-Rabee et al., 2008; Rutstam, 2010). The purpose hiding of constraint is to eliminate Lagrange multipliers in the equation of motion. This paper is an attempt to better understand the system with non-holonomic constraints from the viewpoint of geometric mechanics, which analyzed the problem of motion geometrically (Ariska et al., 2020b, 2019; Gray & Nickel, 2000). The object motion that be discussed in this research is the TT motion. Given the motion of the TT is an example of the motion of objects that can move by translational and rotational (Blankenstein, 2003; Shimomura et al., 2005). In a study of research about TT has successfully solved the equations of reversed TT dynamics in the flat plane while describing the equation of motion using computational physics (Fowles & Cassiday, 2005). Analyzing of complex and complicated dynamic systems that operated on flat and spherical planes at the same time with and without friction has solved well by Poincare Equations.

The dynamics of the TT on the surface in the $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ configuration space with the helping of physics computing. Resolving this equation is not easy, because the configuration space that will be passed by the TT is a cylinder surface which is a curved plane that has cylinder coordinate variables and TT coordinates that move using three coordinate systems, so that the total number of general coordinates to be completed is six common coordinates, i.e. two translational coordinate and three rotational coordinates (Ariska et al., 2018; 2020b). In addition, researchers also analyzed and predicted TT movements with friction by graphic simulation in the real time.

The problem solved in this research is how to analyze the TT motion in the computational by used the motion planning network with JI Method. This research applied technology in solving general equations of a motion system in three-dimensional (3D) space. Considering the growing development of science and technology in the world of education, the dynamics of objects that have a configuration space that is quite complicated because it consists of translational and rotational motion which is very complicated if solved manually. This research is a solution for lecturers and students in completing complex dynamics of objects thoroughly and precisely. This study aims to analyze the dynamics of the mechanical system on the TT with non-holonomic constraints that doing dynamics in the $\mathbb{R} \times S^1 \times SO(3)$ configuration space using physics computing.

The TT motion equation by applying group theory in the form of a rotational group using the Poincare equation in the flat plane has been formulated with physics computation (Broer et al., 2001; Zobova, 2012). In addition, previous studies on the dynamics of mechanical systems have only been formulated for TT that operates in the flat plane. Therefore, the authors are interested in continuing the research by formulating the dynamics of a system that has more complex movements, namely a TT that is played in a curved plane in the form of a surface in a ball with fast and without friction. Detailed motion predictions will be analyzed using physics computation.

2. METHOD

This research is a Computational Physics and math theoretical conducted with a review of reference about geometric mechanic system in the case of the TT that has been developed previously and mathematical calculations using physics computing for rigid body, especially based on Maple. This research is composed from Numerical treatment and present the results of some computations based on the exact system The method used is the motion planning network which can be used for non-holonomic object systems that move in real-time with Jellets Invarian (JI) approach (Rauch-Wojciechowski et al., 2005). In case constants of motion, even if dissipative, has a conserved quantity, we can use Jellets Invarian (JI) approach, $J = -L.q =$ const. where L is the angular momentum of the Tippe Top about the center of mass. We prove this by using Euler equations which govern the evolution of the angular momentum, Straightforward calculations show that Jellet's constant can be written as

$$J = Cn(R\cos(\theta) + A\dot{\eta}R\sin^2\theta) \tag{1}$$

We emphasize once more that the Jellet's constant is an exact constant of motion for the Tippe Top (TT) whether or not there is slipping and independent of the expression for J. It is this constant that to some extent controls the motion of the spinning top. Indeed, it allows a Routhian reduction procedure resulting in relatively simple reduced equations from which we are able to recover in full detail the stability properties of the steady states, Where R defined by,

$$\frac{d}{dt}\frac{\partial R}{\partial v^\rho} - \sum_{\mu=2}^{n}\sum_{\lambda=2}^{n} c^\lambda{}_{\mu\rho} v^\mu \frac{\partial R}{\partial v^\lambda} - \sum_{\mu=2}^{n} c^\lambda{}_{\mu\rho} v^\mu \beta_1 - X_\rho R = 0, \qquad \rho = 2, \ldots, n. \tag{2}$$

This paper used an adaptive method based on second-order numerical differential formulae due to Routhian Reduction (Ariska et al., 2018; Gray & Nickel, 2000) and similar to that described by Jellet Invarian (JI) (Kilin & Pivovarova, 2020; Rauch-Wojciechowski et al., 2005). This method has two desirable properties: easy implementation of time-step adaptivity and allowance for possible stiffness of the system. For most runs, we used a relative error tolerance 10^{-9} (Ariska et al., 2018; 2019; 2020a, 2020b). The computation used is Maple 18. The dynamics of TT are clearly illustrated by Maple 18.

3. RESULTS AND DISCUSSION

Calculation of the TT Constraint is solved by the Jellets Invarian (JI) method. Constraints are conditions that limit the movement of a mechanical system reducing both the degrees of freedom and the range of each degree of freedom (Johnson et al., 2020). Schematic dynamics TT can be seen in Figure 1.

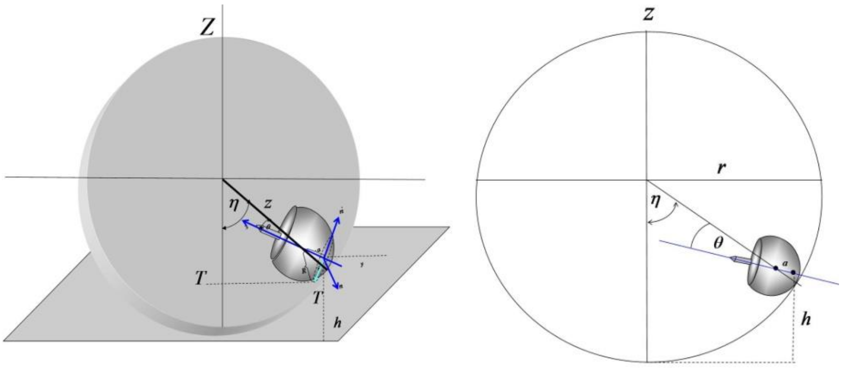

Figure 1 TT Dynamic Schematic in $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$

3.1. Vector Fields of TT in Configuration Space $\mathbb{R} \times \boldsymbol{S^1} \times \boldsymbol{SO(3)}$

The transformation of the rotation operator from $\mathbb{R}^2 \times SO(3)$ to $\mathbb{R} \times S^1 \times SO(3)$ can be written as,

$$\begin{aligned} R_\theta &= (R_x r)\frac{\partial}{\partial r} + (R_x \eta)\frac{\partial}{\partial \eta} + (R_x y)\frac{\partial}{\partial y} \\ &= y\cos\eta\frac{\partial}{\partial r} - \frac{y}{r}\sin\eta\frac{\partial}{\partial \eta} - r\cos\eta\frac{\partial}{\partial y} \end{aligned} \quad (3)$$

$$\begin{aligned} R_\phi &= \left(R_y r\right)\frac{\partial}{\partial r} + \left(R_y \eta\right)\frac{\partial}{\partial \eta} + \left(R_y y\right)\frac{\partial}{\partial y} \\ &= \frac{\partial}{\partial \eta} \end{aligned} \quad (4)$$

$$\begin{aligned} R_\psi &= (R_z r)\frac{\partial}{\partial r} + (R_z \phi)\frac{\partial}{\partial \phi} + (R_z \theta)\frac{\partial}{\partial \theta} \\ &= -y\sin\eta\frac{\partial}{\partial r} - \frac{y}{r}\cos\eta\frac{\partial}{\partial \eta} + r\sin\eta\frac{\partial}{\partial y} \end{aligned} \quad (5)$$

The system $\mathrm{SO}(3)$ is reduced using the Routhian procedure, so the Lagrangian system is,

$$\begin{aligned} L'(\dot\theta, \dot\phi, \dot\psi, \theta) &= T(\dot\theta, \dot\phi, \dot\psi) - U(\theta) \\ &= \frac{1}{2}\left(m(\dot x^2 + \dot y^2) + I\dot\theta^2 + I\sin^2\theta\dot\phi^2 + I_3(\dot\psi + \dot\phi\cos\theta)^2\right) - mg(R - a\cos\theta) \end{aligned} \quad (6)$$

and the cyclic velocity value $\dot\psi$ is

$$\dot\psi = \frac{J - I\dot\phi R\sin^2\theta - I_3\dot\phi\cos\theta((R\cos\theta - a)}{I_3(R\cos\theta - a)} \quad (7)$$

Now, β_1 is defined which is the integration constant in the integral result of equation (2) which has a value in the form of a constant, namely

$$\beta_1 = \frac{J - I\dot\phi R\sin^2\theta}{(R\cos\theta - a)} \quad (8)$$

so, due to the existence of this constant of motion the kinetic energy equation is reduced to

$$T = \frac{1}{2}\left(I\dot\theta^2 + I\sin^2\theta\,\dot\phi^2 + \frac{(\beta_1)^2}{I_3}\right) \quad (9)$$

while the cyclic velocity becomes,

$$\dot{\psi} = \frac{\beta_1}{I_3} - \dot{\phi}\cos\theta \qquad (10)$$

This study analyzes the motion of the TT with low energy with the intention that the TT only moves on the arena in the form of a surface in the tube which is considered quite large than the size of the TT, and the study carried out is classically non-relativistic. So that it is obtained by physical computation based on Maple 18 that the JI equation is constrained for TT moving in the configuration space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ with low energy is,

$$\begin{bmatrix} \ddot{\eta} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} m(a\sin\eta\cos\eta\left(r\sin\theta\,\ddot{\theta} + (r\cos\theta + a\sin^2\theta)\dot{\theta}^2\right)\mathrm{F}_N \\ \left(\frac{1}{I\sin\theta}\right)\left(\frac{S_\phi}{\sin\theta} - 2I\cos\theta\,\dot{\theta}\dot{\phi} - \beta_1\dot{\theta}(1-\csc\theta) + mga\sin\phi\sin\eta\right) \\ ar\sin\theta\,\dot{\eta}^2\cos(2\eta) + a^2\cos\eta\left(\sin^2\theta\,\ddot{\theta} + \sin\theta\cos\theta\,\dot{\theta}^2\right) \\ \beta_1\dot{\theta} - 2I\cos\theta\,\dot{\theta}\dot{\phi}\mathrm{F}_N - \frac{\dot{\theta}\beta_1}{\sin\theta} + mga\sin\phi\sin\eta \\ r^2\ddot{\eta} + a\sin\eta\cos\eta\left(r\sin\theta\,\ddot{\theta} + (r\cos\theta + a\sin^2\theta)\dot{\theta}^2\right) \end{bmatrix} \qquad (11)$$

with an outer variable,

$$|\mathrm{F}_N| = mg\cos\eta + m\ddot{z} + mr\dot{\eta}^2 \;= m\left[g\cos\eta + a\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right) + r\dot{\eta}^2\right] \qquad (12)$$

Initial sate of TT and initial conditions of TT's dynamics in configuration space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ are given Table 1 and 2, respectively. Numerical solution of the equations of the reverse motion for coordinates $\theta(t), \phi(t), \dot{\theta}(t), \dot{\phi}(t), \dot{\psi}(t), \dot{\eta}(t), \dot{X}(t)$with the TT's initial states is in the Table 3.

Table 1. Initial state of TT

I I_n=$I_{n'}$=I (gr.cm^2)	I_3 (gr.cm^2)	m_{total} (g)	*R* (cm)	*D* (cm)
45	50	13	1.3	2.6

The value of initial conditions based on $\eta(0) = 0$ with $\theta(0)$ various, and $\dot{\theta}(0) = \dot{\phi}(0) = \dot{\psi}\,(0) = \dot{\eta}(0) = \dot{X}(0) =\; 0$ can be seen on Table 4.

Table 2. Initial Conditions of TT's Dynamics in Configuration Space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$

No	$\theta(t)$ rad	β_1 $\mathrm{gm^2rad/s}$	μ	Time at reversed second (s)
1	0.1	2,500	0.3	25
2	0.2	2,500	0.3	22
3	0.3	2,500	0.3	Not reversed
4	0.4	2,500	0.3	Not reversed
5	0.5	2,500	0.3	Not reversed
6	0.6	2,500	0.3	Not reversed
7	0.8	2,500	0.3	Not reversed
8	1	2,500	0.3	Not reversed
9	π/2	2,500	0.3	Not reversed

Based on Table 2, it can be seen that the dynamics of TT on the $\mathbb{R} \times S^1 \times SO(3)$ are more random than TT on the $\mathbb{R}^2 \times SO(3)$ in Table 2 and the reversal process requires a more random and irregular time, namely 25 s and 22 s with an elevation angle at the start of TT rotated differently. Likewise, the elevation angle of the rod when the TT is rotated also has differences between the TT on the inner surface of the tube and on the flat plane. The dynamic of TT in the configuration space $\mathbb{R}^2 \times SO(3)$, the limit of the elevation of the rod when it is rotated until TT reverses is $\theta\ (0)\ =\ 0.9$ rad. Whereas the elevation angle limit of the TT's rod that moves in the configuration space $\mathbb{R}^2 \times S^1 \times SO(3)$is if $\eta\ (0)\ =\ 0$,then elevation angle limit $\theta\ (0)\ =\ 0.2$ rad, and if $\eta\ (0)\ =\ \pi\ /\ 2$ the elevation angle limit $\theta\ (0)\ =\ 0,2$ rad. If the initial elevation angle when the TT is rotated is more than 0.2 rad when $\eta\ (0)\ =\ 0$ and 0.4 rad when η (0) = π / 2, then the TT could not be reversed completely using the rod. This occurs due to the different configuration space between $\mathbb{R}^2 \times SO(3)$and $\mathbb{R}^2 \times S^1 \times SO(3)$,the flat surface uses Cartesian coordinates to get the equations of motion while the inner surface of the tube uses the coordinates of the tube to determine the equations of motion. The dynamics of TT will be more varied when moving on the inner surface of the tube because the coordinates in the two configuration spaces are different. The graph obtained by computation Maple 18 based on Table 2 can be seen in Figure 3.

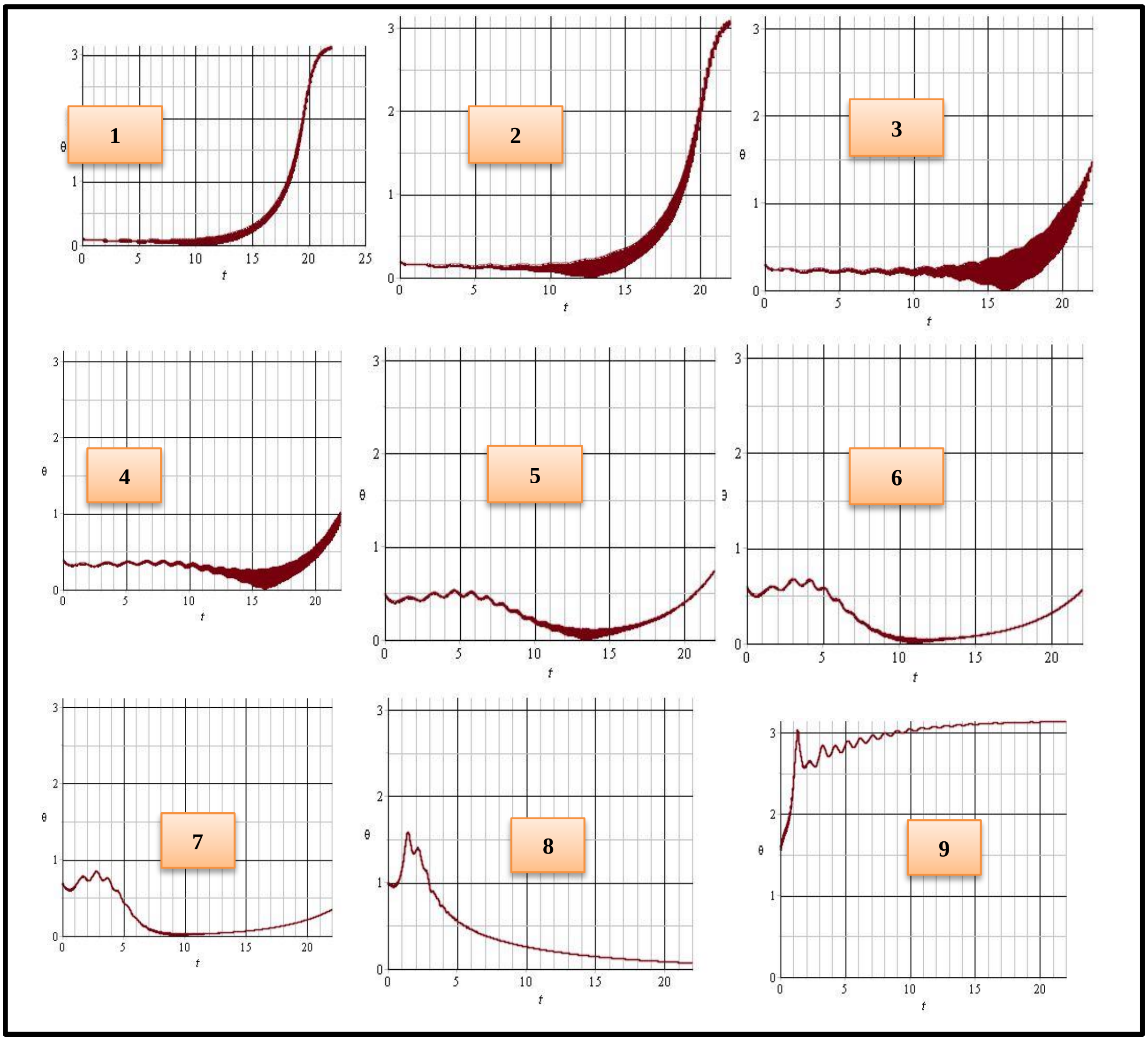

Figure 2. Dynamics of TT in $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$when $\eta(0) = 0$ with value of $\theta(0)$ heterogeneous

Meanwhile, if the value of the initial condition of the TT moving in the configuration room $\mathbb{R} \times S^1 \times SO(3)$ is changed with the elevation angle $\eta\ (0)\ =\ \pi\ /\ 2$ with $\theta\ (0)$ varies with the initial conditions specified in the Table 3, it will be obtained that the computational results of the dynamics of TT are different from the initial conditions in Table 3.

Table 3. Initial Conditions of TT's Dynamic.

No	$\boldsymbol{\theta(t)}$ **rad**	$\boldsymbol{\beta_1}$ $\mathbf{gm^2 rad/s}$	$\boldsymbol{\mu}$	**Time at reversed second (s)**
1	0.1	2,500	0.3	23
2	0.2	2,500	0.3	19
3	0.3	2,500	0.3	20
4	0.4	2,500	0.3	22
5	0.5	2,500	0.3	Not reversed
6	0.6	2,500	0.3	Not reversed
7	0.8	2,500	0.3	Not reversed
8	1	2,500	0.3	Not reversed
9	π/2	2,500	0.3	Not reversed

Whereas the elevation angle limit of the TT's rod that moves in the configuration space $\mathbb{R}^2 \times S^1 \times SO(3)$ is if $\eta\ (0) = \pi / 2$ with $\theta\ (0)$ varies, then elevation angle limit $\theta\ (0) = 0.4$ rad, If the initial elevation angle when the TT is rotated is more than 0.4 rad when η (0) = π / 2, then the TT could not be reversed completely using the rod. Based on Table 1 and Table 3, there are differences in computation results between the TT moving on the surface in the tube $\mathbb{R} \times S^1 \times SO(3)$, if given different initial conditions for elevation angle η (elevation angle when the TT rotates is different). This difference occurs because the angle $\eta(0) = 0$ for Table 4 while $\eta\ (0) = \pi/2$ in Table 3. So, the initial condition when the TT is played determines the length of time the TT will be reversed with the stem. The graph obtained by computation Maple 18 based on Table 3 can be seen in Figure 3.

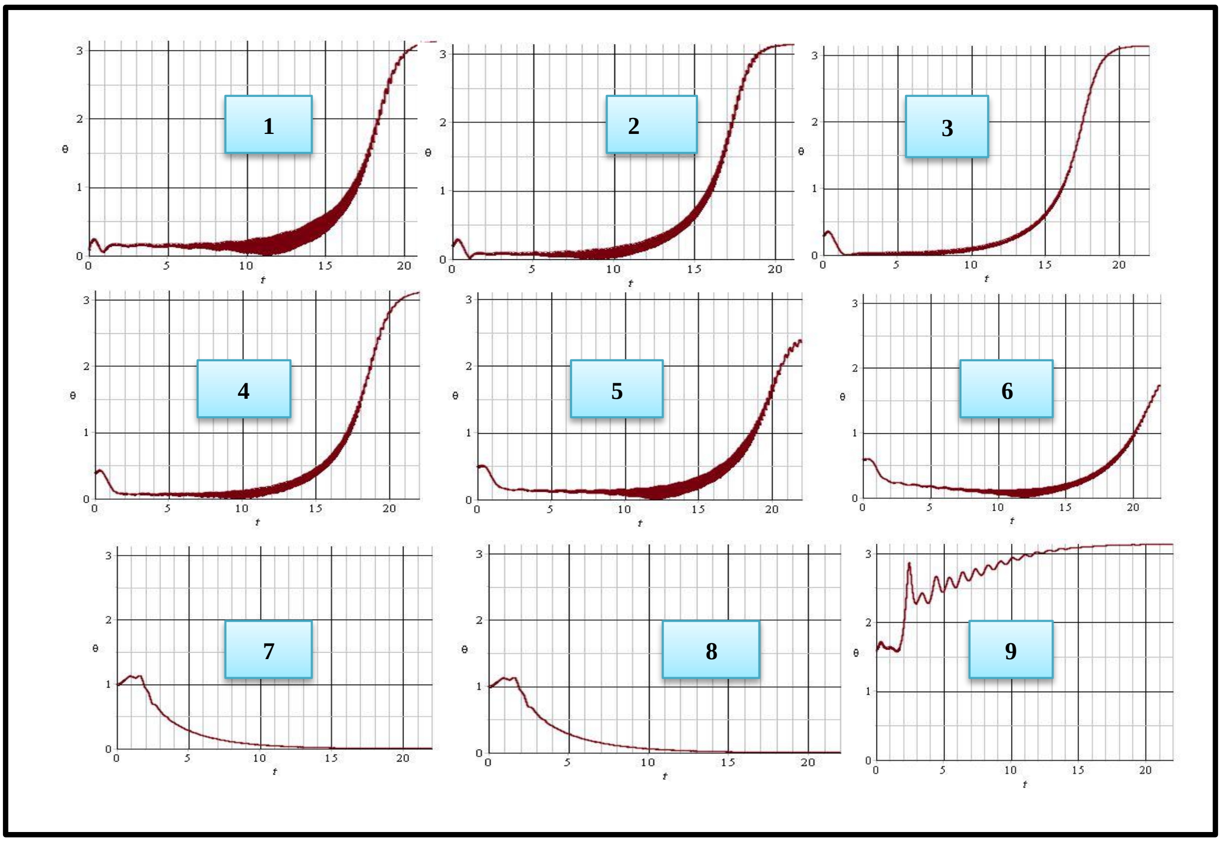

Figure 3. Dynamics of TT in $\mathbb{R} \times S^1 \times SO(3)$ when $\eta(0) = \pi/2$ with value of $\theta(0)$ heterogeneous.

Figure 2 and Figure 3 are the numerical results obtained by researchers in predicting the dynamics of TT in the $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ configuration space. This result is a continuation of research that has been developed previously by various studies including Branicki et al. (2006), Branicki & Shimomura (2006), Kilin & Pivovarova (2020), Rutstam (2010) and Tembely et al. (2019), which have formulated TT's dynamics with numerical solutions with various approaches on the flat surface. The researcher continues by simulating the dynamics of the TT in the curved plane, namely the configuration space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$. This research examines the dynamics of objects at low energy. TT was only analyzed to move in an arena in the form of an $\mathbb{R}^2 \times \mathrm{SO}(3)$ and $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ (flat plane and inner surface of the tube) which is considered large enough from the size of the TT and the study carried out was classical non-relativistic.

4. CONCLUSION

Mechanical systems with a non-holonomic constrains in configuration space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ can be described by Jellet Invarian (JI) reduction, which is a system of rigid body dynamics that can be finished with differential equations and system energy as clearly stated and steady. Based on the results of the previous studies on similar object, the TT were solved by the eular-lagrange equation, Routhian Reduction and Poincare Equations. But, in this study the dynamics of the TT dynamics are successfully solved by the Jellet Invarian (JI) which is Routhian Reduction differential derived from the with the assistance of maple-based computation in configuration space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$. The author succeeded in describing the dynamics of TT motions in real time with the initial conditions that have been finished with several coordinate, especially for cylinder coordinate. The dynamics of TT in configuration space $\mathbb{R} \times \mathrm{S}^1 \times \mathrm{SO}(3)$ can be illustrated using a graph by predicting the motion of the TT with various elevation angles when it is initially rotated with computational assistance. This article can increase knowledge about geometric mechanics theory and its application in analyzing mechanical systems for the design of an appropriate TT playground and understanding the complex motion that exist in nature. This computing physics is useful in describing complex motion in the field of robotics and can be a reference for understanding the non-holonomic constraints in mechanical technology, especially robotics.

REFERENCE

Ariska, M., Akhsan, H., & Muslim, M. (2020a). Dynamic Analysis of Tippe Top on Cylinder's Inner Surface with and Without Friction based on Routh Reduction. Journal of Physics: Conference Series, 1467(1). https://doi.org/10.1088/1742-6596/1467/1/012040

Ariska, M., Akhsan, H., & Muslim, M. (2020b). Potential energy of mechanical system dynamics with nonholonomic constraints on the cylinder configuration space. Journal of Physics: Conference Series, 1480(1). https://doi.org/10.1088/1742-6596/1480/1/012075

Ariska, M., Akhsan, H., & Muslim, M. (2019). Utilization of physics computation based on maple in determining the dynamics of tippe top. Journal of Physics: Conference Series, 1166(1). https://doi.org/10.1088/1742-6596/1166/1/012009

Ariska, Melly, Akhsan, H., & Zulherman, Z. (2018). Utilization of Maple-based Physics Computation in Determining the Dynamics of Tippe Top. Jurnal Penelitian Fisika Dan Aplikasinya (JPFA), 8(2), 123. https://doi.org/10.26740/jpfa.v8n2.p123-131

Blankenstein, G. (2003). Symmetries and locomotion of a 2D mechanical network: the Snakeboard. Lecture Notes for the Euron/GeoPleX Summer School. Bertinoro, Italy.

Bou-Rabee, N. M., Marsden, J. E., & Romero, L. A. (2004). Tippe top inversion as a dissipation-induced instability. SIAM Journal on Applied Dynamical Systems, 3(3), 352–377. https://doi.org/10.1137/030601351

Bou-Rabee, N. M., Marsden, J. E., & Romero, L. A. (2008). Dissipation-induced heteroclinic orbits in tippe tops. SIAM Review, 50(2), 325–344. https://doi.org/10.1137/080716177

Branicki, M., Moffatt, H. K., & Shimomura, Y. (2006). Dynamics of an axisymmetric body spinning on a horizontal surface. III. Geometry of steady state structures for convex bodies. Proceedings of the Royal

Society A: Mathematical, Physical and Engineering Sciences, 462(2066), 371–390. https://doi.org/10.1098/rspa.2005.1586

Branicki, M., & Shimomura, Y. (2006). Dynamics of an axisymmetric body spinning on a horizontal surface. IV. Stability of steady spin states and the "rising egg" phenomenon for convex axisymmetric bodies. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 462(2075), 3253–3275. https://doi.org/10.1098/rspa.2006.1727

Broer, H. W., Krauskopf, B., & Vegter, G. (Eds.). (2001). Global Analysis of Dynamical Systems: Festschrift Dedicated to Floris Takens for His 60th Birthday. CRC Press.

Ciocci, M. C., & Langerock, B. (2007). Dynamics of the tippe top via Routhian reduction. Regular and Chaotic Dynamics, 12(6), 602–614. https://doi.org/10.1134/S1560354707060032

Ciocci, M. C., Malengier, B., Langerock, B., & Grimonprez, B. (2012). Towards a prototype of a spherical tippe top. Journal of Applied Mathematics, 2012. https://doi.org/10.1155/2012/268537

Fokker, A. D. (1952). The tracks of tops' pegs on the floor. Physica, 18(8–9), 497–502. https://doi.org/10.1016/S0031-8914(52)80050-3

Fowles, G. R., & Cassiday, G. L. (2005). Analytical mechanics 7th Ed (Belmont: Thomson Brooks/Cole).

Glad, S. T., Petersson, D., & Rauch-Wojciechowski, S. (2007). Phase space of rolling solutions of the tippe top. Symmetry, Integrability and Geometry: Methods and Applications (SIGMA), 3. https://doi.org/10.3842/SIGMA.2007.041

Gray, C. G., & Nickel, B. G. (2000). Constants of the motion for nonslipping tippe tops and other tops with round pegs. American Journal of Physics, 68(9), 821–828. https://doi.org/10.1119/1.1302299

Holm, D. D. (2011). Geometric mechanics-Part I: Dynamics and symmetry. World Scientific Publishing Company.

Holm, D. D., Schmah, T., & Stoica, C. (2009). Geometric mechanics and symmetry: from finite to infinite dimensions (Vol. 12). Oxford University Press.

Jellett, J. H. (1872). A Treatise on the Theory of Friction. Hodges, Foster.

Johnson, J. J., Li, L., Liu, F., Qureshi, A. H., & Yip, M. C. (2020). Dynamically Constrained Motion Planning Networks for Non-Holonomic Robots. http://arxiv.org/abs/2008.05112

Kilin, A. A., & Pivovarova, E. N. (2020). The influence of the rolling resistance model on tippe top inversion. February. http://arxiv.org/abs/2002.06335

Lewis, A. D. (2017). The physical foundations of geometric mechanics. Journal of Geometric Mechanics, 9(4), 411–437. https://doi.org/10.3934/jgm.2017019

Routh, E. J. (1884). A Treatise on the Dynamics of a System of Rigid Bodies. With Numerous Examples: The advanced part (Vol. 2). Macmillan.

Rauch-Wojciechowski, S., Sköldstam, M., & Glad, T. (2005). Mathematical analysis of the tippe top. Regular and Chaotic Dynamics, 10(4), 333–362. https://doi.org/10.1070/RD2005v010n04ABEH000319

Rutstam, N. (2010). Study of equations for tippe top and related rigid bodies (Doctoral dissertation, Linköping University Electronic Press).

Smale, S. (1970). Topology and mechanics. I. Inventiones mathematicae, 10(4), 305-331.

Shimomura, Y., Branicki, M., & Moffatt, H. K. (2005). Dynamics of an axisymmetric body spinning on a horizontal surface. II. Self-induced jumping. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 461(2058), 1753–1774. https://doi.org/10.1098/rspa.2004.1429

Sriyanti, I., Ariska, M., Cahyati, N., & Jauhari, J. (2020). Moment of inertia analysis of rigid bodies using a smartphone magnetometer. Physics Education, 55(1). https://doi.org/10.1088/1361-6552/ab58ba

Tembely, M., Vadillo, D., Soucemarianadin, A., & Dolatabadi, A. (2019). Numerical simulations of polymer solution droplet impact on surfaces of different wettabilities. Processes, 7(11), 798.

Zobova, A. A. (2011). Different models of friction in double-spherical tippe-top dynamics. In IUTAM Symposium on Dynamics Modeling and Interaction Control in Virtual and Real Environments (pp. 265-272). Springer, Dordrecht.

Zobova, A. A. (2012). Comments on the Paper by M.C. Ciocci, B. Malengier, B. Langerock, and B. Grimonprez "Towards a Prototype of a Spherical Tippe Top." Regular and Chaotic Dynamics, 17(3–4), 367–369. https://doi.org/10.1134/S1560354712030112